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THE CONTRIBUTION OF DIVERGENT WIND COMPONENTS TO THE ENERGY EXCHANGE BETWEEN THE BAROCLINIC AND BAROTROPIC COMPONENTS*

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ABSTRACT

The contribution from the divergent part of the horizontal wind to the energy conversion between the vertical shear flow and the vertical mean flow has been computed using atmospheric data from the isobaric surfaces: 850, 700, 500, 300, and 200 mb. The new calculations supplement earlier computations giving the energy conversion based on an assumption that the horizontal winds are non-divergent.

It is found that the contribution from the divergent part of the horizontal wind normally is very small compared with the contribution from the non-divergent part. The former energy conversion is as a matter of fact generally not significantly different from zero.

The abnormal winter 1962-63 has been investigated separately. It is found that energy conversion by the divergent wind component during this period was much larger and constituted a larger fraction of the total conversion than during any other period.

1. INTRODUCTION

The present study reports on calculations of the contributions from the divergent part of the horizontal wind to the energy conversion between the vertical shear flow and the vertical mean flow. We have recently published (Wiin-Nielsen and Drake [5]) the results of a study of the same energy conversion based on the assumption that the horizontal wind is non-divergent. It has been shown earlier (Wiin-Nielsen [3]) that the total energy conversion between the vertical shear flow and the vertical mean flow can be written as a sum of two contributions of which the first would be present in a quasi-non-divergent model while the second would be excluded in such a model but would be present in a model based on the primitive equations.

The energy conversion which we are concerned with in this paper requires a knowledge of the horizontal divergence in the atmosphere or, alternatively, of the vertical velocity in a coordinate system with pressure as the vertical coordinate. Ideally, we would also require a knowledge of the observed horizontal wind. We have not had access to analyses of the horizontal wind field during the investigated periods, and it has therefore been necessary to make certain approximations which will be explained in the following section.

A pilot calculation of the energy conversion in question was performed in the paper by Wiin-Nielsen [3]. This calculation was based on a minimum vertical resolution using only two isobaric surfaces (850 and 500 mb.) and a single vertical velocity at 600 mb. One of the main purposes of this study is to extend the pilot calculation to a greater vertical resolution and to larger time periods than a single winter month.

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It should be pointed out that the energy conversion described in this paper can be compared directly with the pilot calculation in [3]. Our calculations are, however, not directly comparable to any of the energy quantities described by Smagorinsky [2] simply because he only computes the total energy conversion between shear flow and mean flow. A calculation of the kinetic energy conversions used by Smagorinsky [2] from atmospheric data will be presented in a later paper.

The top level in this calculation is the 200-mb. surface. Since this surface is in the troposphere over a major part of the Northern Hemisphere, we have not included stratospheric data to any major extent in our calculations.

2. FORMULATION OF THE CALCULATIONS

Since the basic derivations were given in [3] it will not be necessary to give them here. It suffices to state that the energy conversion which we want to compute is expressed by the integral

$$C_D(K_S, K_M) = -\frac{1}{2\pi g (\sin \varphi_2 - \sin \varphi_1)} \int_0^{p_0} \int_{\varphi_1}^{\varphi_2} \int_0^{2\pi} (\mathbf{v}_M \cdot \mathbf{v}_S) \nabla \cdot \mathbf{v}_S \cos \varphi d\lambda d\varphi dp. \quad (2.1)$$

The symbols appearing in (2.1) have the following meaning: K_M is the kinetic energy of the vertically averaged flow, K_S is the kinetic energy of the vertical shear flow, \mathbf{v} is the horizontal wind vector, g is gravity, λ is longitude, φ is latitude, and p is pressure. A subscript M refers the vertical mean flow, while a subscript S refers to the vertical shear flow. The vertical average is defined by the integral

$$(\)_M = \frac{1}{p_0} \int_0^{p_0} (\) dp \quad (2.2)$$

where p_0 is a standard value of the surface pressure. The subscript S is then defined by the relation:

$$(\)_S = (\) - (\)_M. \quad (2.3)$$

The energy conversion symbol $C_D(K_S, K_M)$ means the conversion by the divergent part of the wind (subscript D) from the kinetic energy, K_S , of the shear flow to the kinetic energy, K_M , of the vertical mean flow.

The integral (2.1) is evaluated over a region between the two latitude circles φ_1 and φ_2 and between the top of the atmosphere, $p=0$, to the ground, $p=p_0$. It should furthermore be pointed out that (2.1) is written as an energy conversion per unit area. The unit of $C_D(K_S, K_M)$ is therefore in the MTS-system: $\text{kJ.m.}^{-2} \text{sec.}^{-1}$.

The major problem in evaluating the integrand in (2.1) is to obtain the divergence, $\nabla \cdot \mathbf{v}_S$. The divergence of the vertically averaged flow, $\nabla \cdot \mathbf{v}_M$, is zero in our calculations simply because we have assumed that the vertical velocity $\omega=0$ for $p=p_0$. We have therefore that $\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{v}_S$. The divergence, $\nabla \cdot \mathbf{v}$, has in this calculation been obtained

from a knowledge of the vertical velocity through the use of the continuity equation:

$$\nabla \cdot \mathbf{v} = -\frac{\partial \omega}{\partial p}. \quad (2.4)$$

The vertical velocities were, in turn, obtained from a solution of the so-called ω -equation which is derived from the vorticity equation and the thermodynamic equation by elimination of the time derivatives. We have used the vorticity equation in the following simple, but consistent, form:

$$\frac{\partial \nabla^2 \psi}{\partial t} + \mathbf{v} \cdot \nabla (\zeta + f) = f_0 \frac{\partial \omega}{\partial p}, \quad (2.5)$$

where ψ is the stream function, f the Coriolis parameter, f_0 a standard value, $\zeta = \nabla^2 \psi$ the vorticity, while the other symbols have been defined earlier. We note that the vertical advection of vorticity and the term expressing the turning of the vortex tubes (the so-called "twisting" or "tipping" term) have been neglected, as well as all reference to friction.

The thermodynamic equation was used in its adiabatic form

$$\frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial p} \right) + \mathbf{v} \cdot \nabla \left(\frac{\partial \psi}{\partial p} \right) + \frac{1}{f_0} \sigma \omega = 0. \quad (2.6)$$

in which it has been assumed that $\partial \psi / \partial p = f_0^{-1} \partial \phi / \partial p$. For the justification of this assumption, see Phillips [1]. $\sigma = -\alpha \partial \ln \theta / \partial p$ is a measure of static stability, and it has for consistency been assumed that σ is a function of pressure only, $\sigma = \sigma(p)$.

By differentiation of (2.5) with respect to pressure and by applying the Laplacian operator to (2.6) we obtain after subtraction the ω -equation:

$$\sigma \nabla^2 \omega + f_0^2 \frac{\partial^2 \omega}{\partial p^2} = f_0 \left\{ \frac{\partial}{\partial p} [\mathbf{v} \cdot \nabla (\zeta + f)] - \nabla^2 \left[\mathbf{v} \cdot \nabla \frac{\partial \psi}{\partial p} \right] \right\}, \quad (2.7)$$

which is the equation which has to be solved for ω under proper boundary conditions. The horizontal wind appearing on the right-hand side of (2.7) has been approximated by the non-divergent wind $\mathbf{v} = \mathbf{k} \times \nabla \psi$, where \mathbf{k} is a vertical unit vector. The streamfunction, ψ , was determined by the method described in the earlier paper by Wiin-Nielsen and Drake [5], giving as results the streamfunction at the levels 200, 300, 500, 700, and 850 mb. These levels are indicated as odd levels in figure 1. With the streamfunctions at the odd levels in figure 1 it is possible to compute the values of the forcing function (the right-hand side of (2.7)) at the even levels in figure 1 approximating derivatives with respect to pressure by centered finite differences. The left-hand side of equation (2.7) can furthermore be approximated at the even levels ($q=2, 4, 6$, and 8) by centered finite differences using the boundary conditions $\omega=0$ for $p=0$ and $p=p_0$.

It should be noted that the boundary condition at the lower level can be improved by considering the effects of

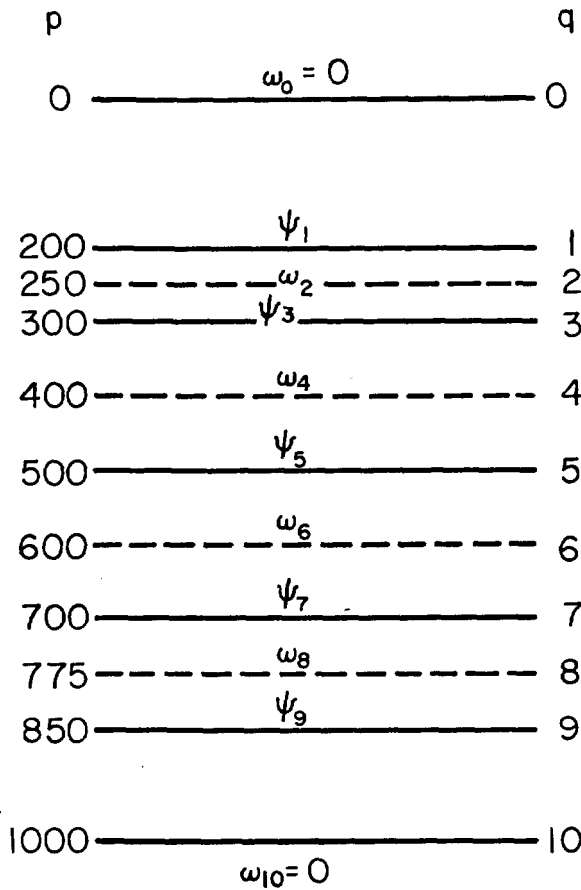


FIGURE 1.—The vertical grid used in the calculations. The symbols indicate the quantities available at the different levels.

mountains and friction. However, in the derivation of the basic formula (2.1) it was assumed that the simplified boundary condition $\omega=0$, $p=p_0$ applies, and we should therefore for consistency use the same condition in the calculation of ω from (2.7). $\sigma=\sigma(p)$ appearing as a coefficient to $\nabla^2\omega$ in (2.7) has values derived from a standard atmosphere. The values of σ are needed at the even levels ($q=2, 4, 6$, and 8) in figure 1. Standard relaxation procedures were used to solve equation (2.7).

The next problem connected with the evaluation of the integrand in (2.1) is the computation of the scalar product $\mathbf{v}_M \cdot \mathbf{v}_S = u_M u_S + v_M v_S$. In the present calculations we have approximated the zonal and meridional wind components by the non-divergent assumption; i.e.

$$u = -\frac{1}{a} \frac{\partial \psi}{\partial \varphi}, \quad v = +\frac{1}{a \cos \varphi} \frac{\partial \psi}{\partial \lambda} \quad (2.8)$$

where a is the radius of the earth. This approximation was used for both the vertically averaged wind \mathbf{v}_M and the vertical shear wind \mathbf{v}_S . From the procedures outlined above it can be seen that our computational procedure can be considered as the first step in an iterative procedure. Having the streamfunction at the odd levels we compute ω at the even levels from (2.7). We could next compute

the divergence $\nabla \cdot \mathbf{v}$ from the continuity equation at the odd levels. However, we have

$$\nabla^2 \chi = \nabla \cdot \mathbf{v} \quad (2.9)$$

where χ is the velocity potential. Solving equation (2.9) at the odd levels, we can obtain the velocity potential and therefore the divergent part of the wind $\mathbf{v}_\chi = \nabla \chi$. These wind components could then be added to the original non-divergent winds. The resulting total wind could then be used to compute a new value of ω from (2.7), etc. We have not used this procedure partly because a test calculation showed no major differences in the vertical velocities, but mainly because we, by using the cyclic calculation, go outside the framework of the quasi-non-divergent model. A more general equation than (2.7) for the vertical velocity should be used in such a case.

After having obtained the vertical velocities from (2.7) we computed the divergence from a finite difference form of (2.4)

$$D_q = (\nabla \cdot \mathbf{v})_q = -\frac{\omega_{q+1} - \omega_{q-1}}{(\Delta p)_q}; \quad q \text{ odd}, \quad (2.10)$$

where q refers to the counter appearing in figure 1. It is now possible to compute the integrand in (2.1) at all the odd levels. We may therefore write:

$$C_D(K_S, K_M) = \sum_q C_q(K_S, K_M); \quad q \text{ odd}, \quad (2.11)$$

where

$$C_q(K_S, K_M) = -\frac{(\Delta p)_q}{2\pi g (\sin \varphi_2 - \sin \varphi_1)} \int_{\varphi_1}^{\varphi_2} \int_0^{2\pi} (\mathbf{v}_M \cdot \mathbf{v}_{S,q}) D_q \cos \varphi d\lambda d\varphi. \quad (2.12)$$

We are next going to express the integral (2.12) in the wave number regime. In order to accomplish this result we make a Fourier analysis of the basic streamfunction data. The procedure which we have followed is copied from our earlier paper [5].

It is seen that the integrand in (2.12) consists of two terms both of which are products of three factors. We are therefore dealing with two integrals of the type which were treated in general in appendix A of Wiin-Nielsen and Drake [5]. The computational method which we have used in expressing (2.12) in the wave number regime:

$$C_q(K_S, K_M) = C_q^{(0)}(K_S, K_M) + \sum_{n=1}^N C_q^{(n)}(K_S, K_M) \quad (2.13)$$

can therefore be obtained without modification from these general formulas.

Before we proceed to describe the results of the computations evaluating (2.12) and (2.13) from actual data it is worth while to consider the physical and kinematical inter-

pretation of (2.1). We note first of all that the velocity vector \mathbf{v}_M in general will be approximately equal to the wind somewhere in the middle troposphere. At the same level we will have $\mathbf{v}_S=0$ by definition. For this reason alone we would expect small contributions to the integral (2.1) from the mid-tropospheric levels. It is furthermore well known that there is a general tendency to have a level of non-divergence, $\nabla \cdot \mathbf{v}_S=0$, in the mid-troposphere. This fact is a second reason to expect small contributions from levels around 500 mb.

We are next going to consider the contribution from the upper and lower parts of the troposphere. If the wind does not turn too much with height we will find that the vectors \mathbf{v}_M and \mathbf{v}_S form an angle less than 90° in the upper troposphere and that $\mathbf{v}_M \cdot \mathbf{v}_S$ therefore will be positive. Under the same conditions we will expect that \mathbf{v}_M and \mathbf{v}_S will tend to oppose each other in the lower levels of the troposphere, and that $\mathbf{v}_M \cdot \mathbf{v}_S$ will be negative at such levels. On the other hand, if we have divergence at the higher levels of the troposphere we will in general have convergence at the lower levels and vice versa. It is thus seen that there will be a general tendency to get contributions of the same sign from the upper and lower parts of the troposphere. The contribution will be negative at locations where we have upper level divergence and lower level convergence, while a positive contribution will be obtained with the opposite arrangement.

The description of the results, given in the next section, will confirm the qualitative reasoning given above.

3. RESULTS OF THE ENERGY CONVERSION CALCULATIONS

The calculations described in section 2 have been carried out for six different months: January, April, July, October, December, 1962, and January 1963. The available data will in general permit us to make two calculations per day, corresponding to the observation times at 00 and 12 GMT. Occasionally, we have had missing data on the magnetic tapes giving the height analyses of the isobaric surfaces, but the percentage of such cases is very small.

We shall first consider the mean values for each of the six months for which calculations have been made. The averaged values are reproduced in table 1 of this paper. For easy reference we have also reproduced the corresponding results for the energy conversion $C_{ND}(K_S, K_M)$ from [5]. It is seen from table 1 that $C_D(K_S, K_M)$ is small compared to $C_{ND}(K_S, K_M)$ for all five months for which calculations have been made during 1962. The percentage varies from month to month but is less than 11 percent during all months in 1962. One would expect this result from the quasi-geostrophic theory and from the fact that this theory has been used for the calculation of the vertical velocities which form the basis of our calculation of $C_D(K_S, K_M)$. One can as a matter of fact consider the present calculation as a test of the validity of the quasi-geostrophic theory because it was shown in the original study [3] that only $C_{ND}(K_S, K_M)$, but not $C_D(K_S, K_M)$,

TABLE 1.—Values of $C_{ND}(K_S, K_M)$, $C_D(K_S, K_M)$, $C(K_S, K_M)$, and C_D/C_{ND} in percent for different months. Unit for the energy conversions is $10^{-4} \text{ kJ. m}^{-2} \text{ sec.}^{-1}$

	$C_{ND}(K_S, K_M)$	$C_D(K_S, K_M)$	$C(K_S, K_M)$	$C_D/C_{ND}(\%)$
January 1962.....	46.5	0.31	46.8	0.7
April 1962.....	28.8	1.66	30.5	5.8
July 1962.....	12.4	0.62	13.0	5.0
October 1962.....	29.6	2.66	32.3	9.0
December 1962.....	43.2	4.76	48.0	11.0
January 1963.....	41.6	14.58	56.2	35.0

makes a contribution to the energy conversion between the vertical shear flow and the vertical mean flow in a quasi-geostrophic model. The results relating to the months from the year 1962 may therefore be considered to indicate that the quasi-geostrophic theory was valid to the 10 percent level of accuracy during this time period.

The results from January 1963 are very different from the others. We find for this month that $C_D(K_S, K_M)$ is more than $\frac{1}{3}$ of $C_{ND}(K_S, K_M)$. One must be careful in drawing conclusions from these numbers. It seems, however, justified to state that the quasi-geostrophic theory is a poor approximation to the atmospheric flow during this period, simply because some of the terms which have been neglected in the quasi-geostrophic theory had an appreciable magnitude on the average during the month of January 1963 even when they are evaluated using results (the vertical velocities) from the quasi-geostrophic theory. It is, on the other hand, difficult to justify that our calculations of $C_D(K_S, K_M)$ measure this energy conversion in a realistic way. We must emphasize that the vertical velocities which play such an important role in our calculations were computed using the quasi-geostrophic theory. Furthermore, we have neglected the influence of diabatic heating and friction in our calculations. It is known that these factors may give significant contributions to the fields of vertical velocity and divergence on the largest scales of atmospheric motions.

The spectra, giving $C_D(K_S, K_M)$ as a function of wave number, for the different months show that the major contribution to it comes from the small wave numbers. These spectra are rather irregular without any distinct maxima and minima. It is therefore not worthwhile to reproduce all of them in this paper. It suffices to give a couple of examples. We have selected the spectra for January 1962 and January 1963, given as figures 2 and 3, respectively. These figures show that we have a large contribution from wave number 0 (the zonal currents), especially in January 1963. It is furthermore seen that wave number 3 gives a negative contribution during both months. However, this is not always the case. The spectra for the other months (not reproduced) show that the contribution from this particular wave number just as often is positive.

It is of some interest to find the contributions from the zonal currents and the eddies separately. This information is given in table 2 for the six months mentioned earlier.

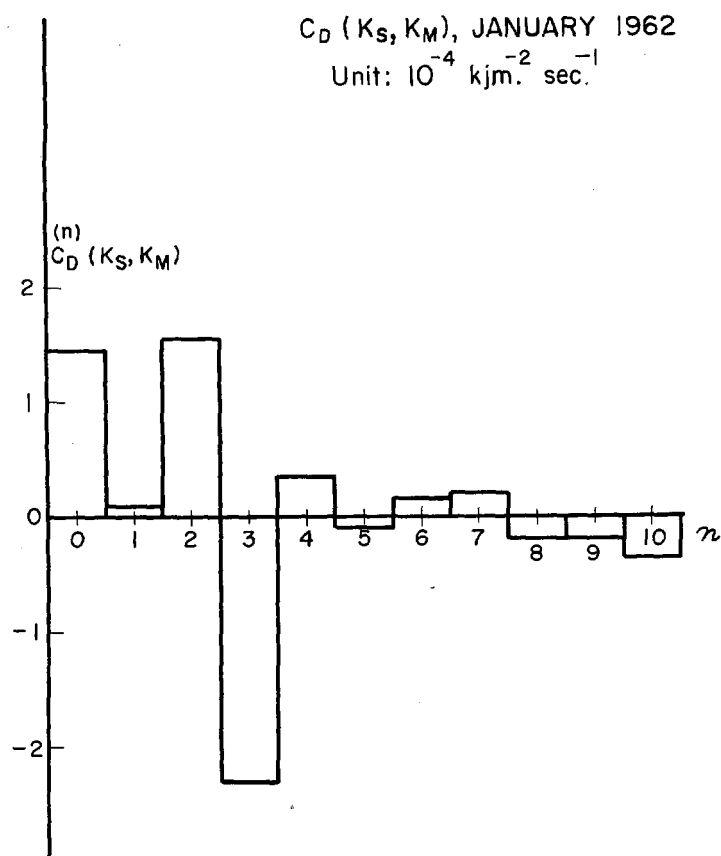


FIGURE 2.—The energy conversion $C_D(K_S, K_M)$ as a function of wave number. The spectrum shows averaged results for January 1962. Unit: $10^{-4} \text{ kJ. m.}^{-2} \text{ sec.}^{-1}$

It is seen that a considerable fraction of the total energy conversion $C_D(K_S, K_M)$ is connected with the zonal currents. As a matter of fact, we find that about $\frac{2}{3}$ of $C_D(K_S, K_M)$ during the month of January 1963 is found in $C_D^{(0)}(K_S, K_M)$.

The mean values which we have calculated for $C_D(K_S, K_M)$ for the different months are small compared to $C_{ND}(K_S, K_M)$ as can be seen from table 1. The only exception in the present sample is January 1963. The statistical significance of the monthly averages can be found by computing the standard deviations. These values are found in table 3.

TABLE 2.—Monthly mean values of $C_D^{(0)}(K_S, K_M)$ and $\Sigma C_D^{(n)}(K_S, K_M)$ for the months indicated. Unit: $10^{-4} \text{ kJ. m.}^{-2} \text{ sec.}^{-1}$

	$C_D^{(0)}(K_S, K_M)$	$\Sigma C_D^{(n)}(K_S, K_M)$	$C_D(K_S, K_M)$
January 1962.....	1.45	-1.14	0.31
April 1962.....	0.72	+0.94	1.66
July 1962.....	0.33	+0.29	0.62
October 1962.....	1.00	+1.66	2.66
December 1962.....	2.75	+2.00	4.75
January 1963.....	9.20	+5.38	14.58

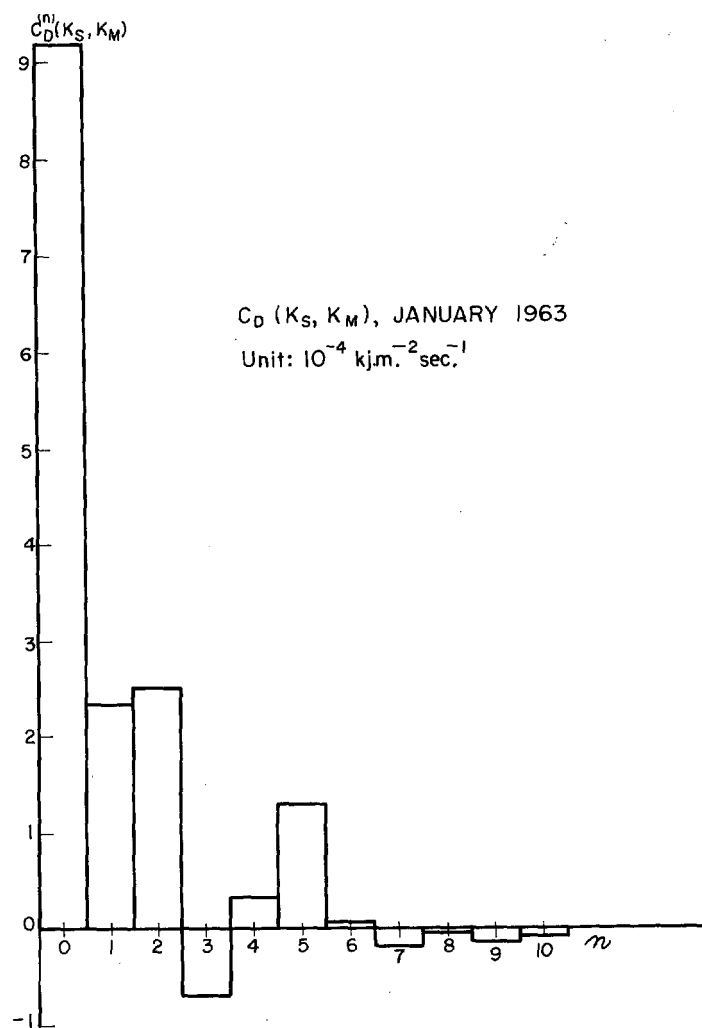


FIGURE 3.—The energy conversion $C_D(K_S, K_M)$ as a function of wave number. The spectrum shows averaged results for January 1963. Unit: $10^{-4} \text{ kJ. m.}^{-2} \text{ sec.}^{-1}$

It is seen that the standard deviations in the cases from the year 1962 are larger than the mean values, a fact which indicates that the mean values are not significantly different from zero. The case of January 1963 is again an exception in which the standard deviation is somewhat smaller than the mean value.

It seems surprising that $C_D(K_S, K_M)$ is so small during January 1962 and is even less than the value for July 1962.

TABLE 3.—Monthly mean values and standard deviations of $C_D(K_S, K_M)$ in the units, $10^{-4} \text{ kJ. m.}^{-2} \text{ sec.}^{-1}$

	Mean Value	Standard Deviation
January 1962.....	0.31	7.3
April 1962.....	1.66	4.0
July 1962.....	0.62	1.6
October 1962.....	2.66	3.7
December 1962.....	4.75	6.0
January 1963.....	14.58	11.3

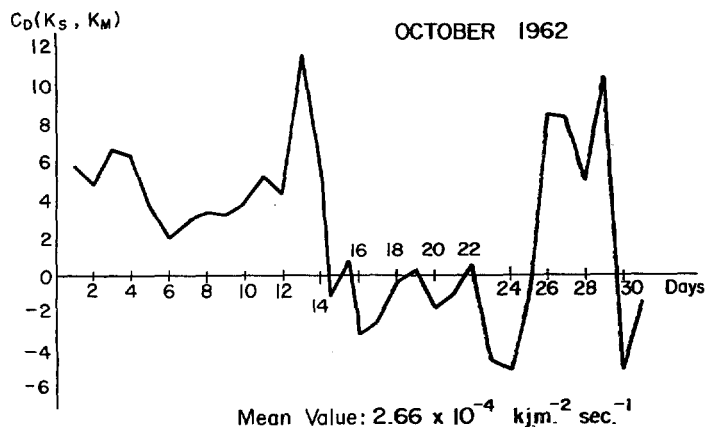


FIGURE 4.—The energy conversion $C_D(K_S, K_M)$ as a function of time for October 1962. The abscissa is days of the month and the ordinate $C_D(K_S, K_M)$ in units of $10^{-4} \text{ kJ. m.}^{-2} \text{ sec.}^{-1}$

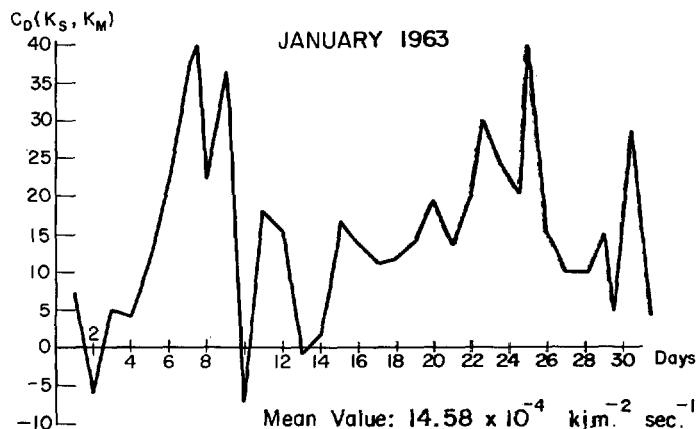


FIGURE 5.—The energy conversion $C_D(K_S, K_M)$ as a function of time for January 1963. Coordinates as in figure 4.

One would expect to find larger values during winter than during summer. However, it should be borne in mind that all the values for the year 1962 are insignificantly different from zero, and that no particular significance can be attached to the numerical values. To give the reader some idea about the variation throughout a month we have reproduced the values of $C_D(K_S, K_M)$ as a function of time for the months of October 1962 and January 1963 in figures 4 and 5, respectively. It is seen that the energy conversion remains of one sign during rather long periods of time. One can furthermore observe that there is considerable variation in the values of the energy conversion from one day to the next.

We shall finally comment on the contributions to $C_D(K_S, K_M)$ from different pressure levels and different latitudes. In figures 6 and 7 we have reproduced these contributions as a function of latitude. It is seen that we have small positive contributions in the very high

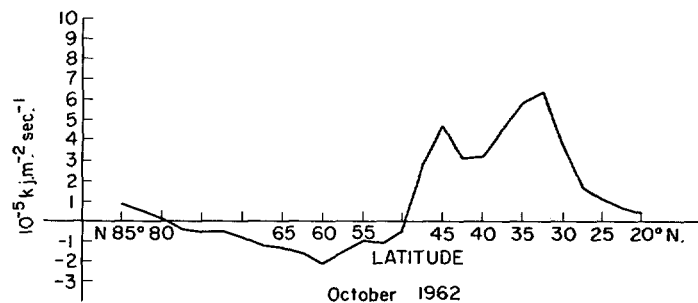


FIGURE 6.—The contribution to $C_D(K_S, K_M)$ as a function of latitude for October 1962. Unit: $10^{-5} \text{ kJ. m.}^{-2} \text{ sec.}^{-1}$

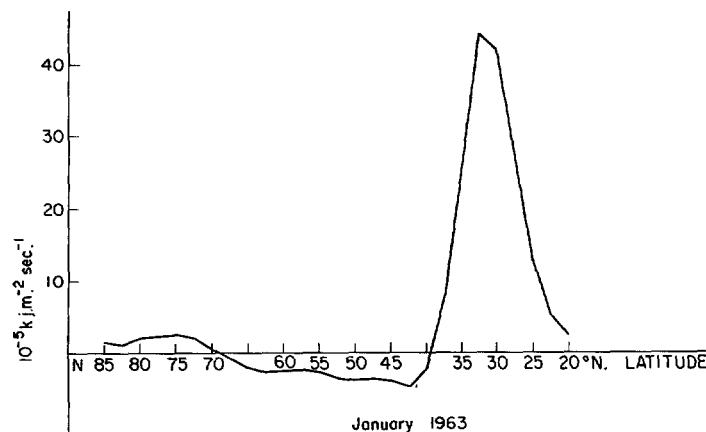


FIGURE 7.—The contribution to $C_D(K_S, K_M)$ as a function of latitude for January 1963. Unit: $10^{-5} \text{ kJ. m.}^{-2} \text{ sec.}^{-1}$

latitudes, negative contributions in a rather broad band of latitudes around 60°N. , and the major positive contribution from the subtropical latitudes. One of the reasons for the very high value in January 1963 (fig. 7) is the large positive contribution centered around 30°N.

An even greater insight into the contributions from the different levels in the atmosphere can be gained from figures 8 and 9 giving the contributions to $C_D(K_S, K_M)$ as a function of latitude and pressure. The tendency to have very small contributions from the mid-troposphere and to have the same sign in the upper and lower parts of the atmosphere can clearly be seen in the two figures.

Combining the results shown in figures 6, 7, 8, and 9 with the reasoning given at the end of section 2 of this paper we can deduce the average position of the major regions of convergence and divergence in the atmosphere. It is seen that we have divergence in the lower levels and convergence in the higher levels in the subtropical latitudes (20°N. to about 40°N. in January 1963, 20°N. to about 50°N. in October 1962). North of this region we find convergence in the lower levels and divergence in the higher levels, while the situation in the very high latitudes is reversed.

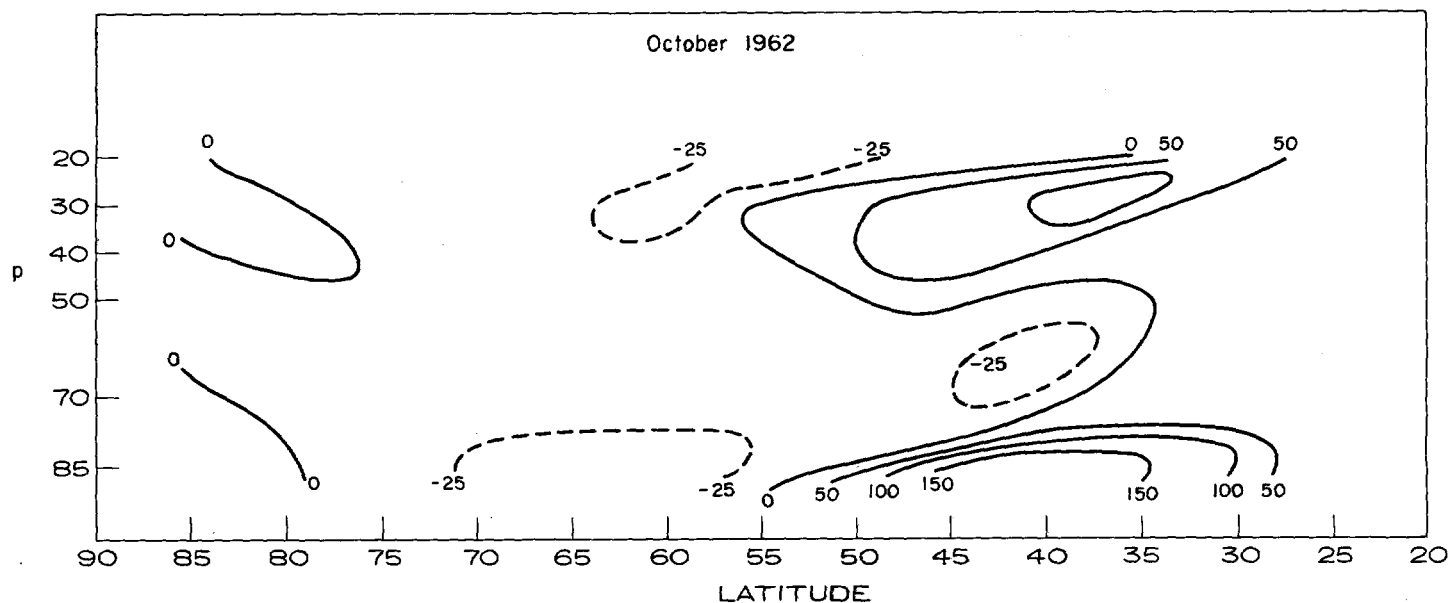


FIGURE 8.—The contribution to $C_D(K_S, K_M)$ as a function of latitude and pressure for October 1962. Unit: $10^{-8} \text{ kJ. m.}^{-2} \text{ sec.}^{-1} \text{ cb.}^{-1}$

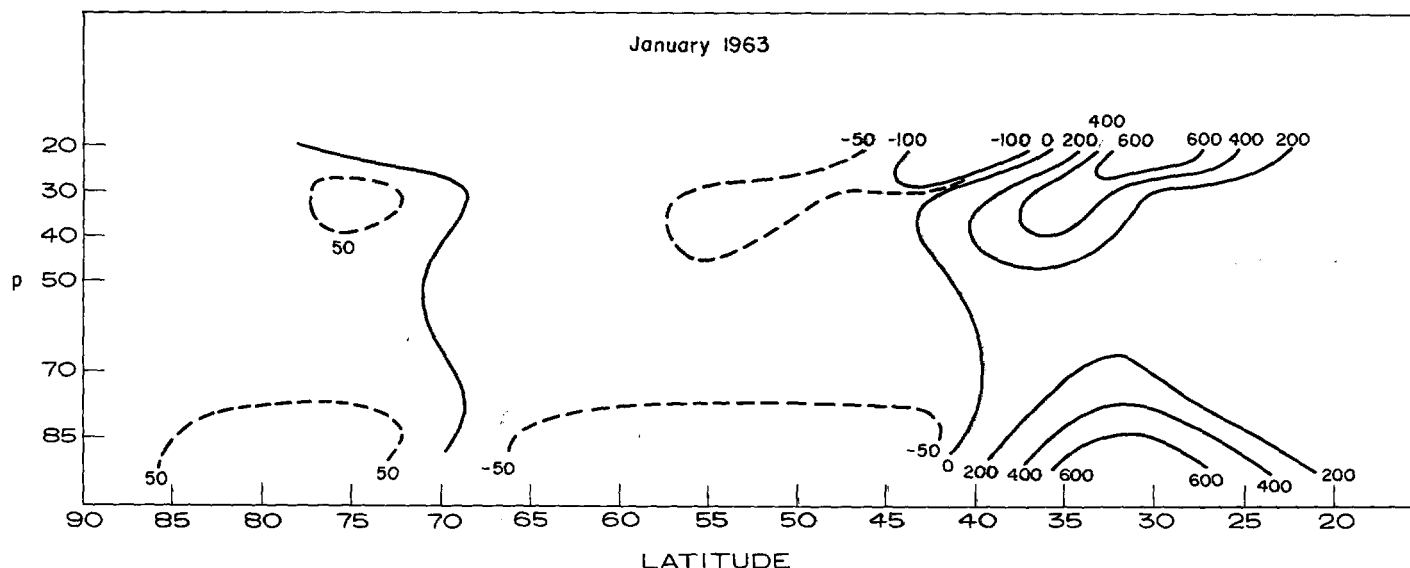


FIGURE 9.—The contribution to $C_D(K_S, K_M)$ as a function of latitude and pressure for January 1963. Unit: $10^{-8} \text{ kJ. m.}^{-2} \text{ sec.}^{-1} \text{ cb.}^{-1}$

4. CONCLUDING REMARKS

The calculations of the divergent part of the energy conversion $C(K_S, K_M)$ show that we normally have a small ratio $C_D(K_S, K_M)/C_{ND}(K_S, K_M)$. This is in agreement with the quasi-geostrophic nature of the atmospheric circulation. Our calculations show furthermore that $C_D(K_S, K_M)$ is not significantly different from zero during most of the time periods which have been considered. The results from January 1963 turn out to be rather different from the other results. This is in agree-

ment with computations of many other energy conversions, in particular the energy conversion from eddy kinetic to zonal kinetic energy described by the authors in [4].

The main uncertainty connected with the calculations described in this paper is the approximations which it is necessary to make in order to compute the vertical velocities. This uncertainty is common in the calculations of several other energy conversions in the atmosphere, in particular the energy conversion from available potential energy to kinetic energy. It is known that the adiabatic,

frictionless vertical velocities computed from equation (2.7) may be in considerable error because of the neglect of diabatic heating and friction. The results obtained in this paper should therefore be considered as a first approximation to the energy conversion $C_D(K_S, K_M)$, and the calculations should be repeated when it is possible to incorporate the neglected effects. Even if our results may have errors resulting from the factors mentioned above, they are nevertheless a measure of the goodness of the quasi-geostrophic theory in its simplest form.

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